MEDIA HIGHLIGHTS

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Media Highlights are short, approximately half-page, reviews intended to help *CMJ* readers monitor a broad spectrum of publications, web materials, professional activities, and instructional resources. Readers are encouraged to submit items that will be of interest to colleagues in the mathematical community. Media Highlights should be sent to Warren Page at wxpny@aol.com.

The Last Boat from Lisbon: Conversations with Peter D. Lax, Istvan Hargittai. *The Mathematical Intelligencer* 23:3 (Fall 2010) 24–30.

With his family, Peter Lax emigrated from Hungary to the United States in 1941, on the last boat to leave Lisbon before the United States entered World War II. He bore a letter from Rozsa Peter to John von Neumann that praised the 15-year-old Lax as her most talented student, and ending, "I would like to see him in good hands out there because I am convinced that he may amount to something." Lax fulfilled her prophecy, becoming one of our most distinguished applied mathematicians, the Director of the Courant Institute, and winning the 2005 Abel Prize "for his groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions." In this interview, he talks candidly about his education, his career, and other great Hungarian-born mathematicians and scientists. Here are a few samples:

- On schools: "In America, the teachers were friends. In the European school, you recognized who your enemy was—the teachers. That could explain why European schools were so efficient. You had to fight for your life." (Rozsa Peter tutored him outside of school.)
- On Paul Halmos' famously-titled article "Applied Mathematics is Bad Mathematics": "The fact is that he knew nothing about it, so whatever he said was irrelevant."
- On Paul Erdős: "Erdős did some very great things, but . . . what I found strange was that he was willing to work on anything. It was partly kindness: when people came to him with problems, he was very willing to do it. And partly it was just that he was interested in everything."
- On the "five Martians" of Hungarian science—Theodore von Karman, Leo Szilard, Eugene Wigner, John von Neumann, and Edward Teller: "Szilard perhaps had the most fantastic imagination. Perhaps he was the most remarkable among them. But von Neumann had a mind which was, in its power, unlike anybody else's."

Lax tells the story of when he was Director of the Computing Center at the Courant Institute at the height of student unrest in 1970, when students occupied the Institute and threatened to destroy its computer unless the University put up $100,000 bail for the Black Panthers. The University didn't give in, and after two days the students left.

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When Lax and his colleagues re-entered the building, they smelled smoke and found a burning fuse leading to bottles of flammable liquids tied to the computer. They defused the bomb and saved the computer. “Afterwards, [my wife] Anneli asked how I could be so crazy to run toward a burning bomb. I told her that I was so angry that I didn’t think.”

The interview ends with this exchange: *On a lighter note, do you count in English or in Hungarian?* “In Hungarian.” *And you curse in Hungarian.* “Sorry about that. I do that out of tact.” **PDS**

The Pattern Collector, Julie Rehmeyer. *Science News’ Math Trek* (August 6, 2010), `http://www.sciencenews.org/view/generic/id/61870`.

This article outlines the personal and professional history of the encyclopedia of integer sequences founded by Neal Sloane. The encyclopedia started with a set of cards bearing sequences Sloane had encountered, evolved through two books, a website, and is now a wiki. Sloane’s favorite sequence, the Recamán sequence, is pointed to as one that might “wreck your life.” He spent months working with it. Seemingly “just a curiosity, unconnected to other mathematical questions,” Sloane views it as a wonderful discovery that “provides a welcome escape from the troubles of our planet.” **NS**

What Is the Probability That the Final Person on the Aircraft Sits in His Own Seat?, Paul Belcher. *Mathematical Spectrum* 42:3 (2009/2010) 107–110.

The following problem has been passed around mathematical circles lately: n passengers board a plane with n seats. The first passenger to board has lost his boarding pass and sits in a seat at random. The other $n - 1$ passengers sit in their seats if they’re unoccupied, otherwise they chose a seat at random. What is the probability that the nth passenger gets his own seat? The surprising answer is one-half, independent of the number n. The article shows this by using a tree diagram for the special-but-suitably-general case when n is 4. The eight paths pair up into four pairs, where both paths in a pair have the same probability. Each pair includes a path where the nth passenger gets his own seat and a path where he doesn’t. So for $n = 4$, the probability that the last passenger gets his own seat must be $\frac{1}{2}$. The article cleverly shows that the pairing generalizes for any n, by looking at the last person among the first $n - 1$ passengers who does not get his own seat. It also shows why the tree diagram for n passengers has exactly 2^{n-1} different paths.

The article gives two additional proofs that the answer is always $\frac{1}{2}$. Both are by mathematical induction, a short one by strong induction and a much longer one by ordinary (or weak) induction. **PR**

The Spread of Behavior in an Online Social Network Experiment, Damon Centola. *Science* 329 (September 2010) 1194–1196.

The author conducted a controlled experimental study of how the topology of a real social network is related to the diffusion of information. Organizing the participants of a community into a clustered lattice network in one set of trials, and into a random network in the other set of trials, Centola explored the time it took for a particular piece of information, in this case a particular health practice, to be acted upon by members of the community. The study also allowed Centola to view the actions of a given actor in the community after receiving the message from multiple other actors. The results show that social reinforcement through multiple, redundant ties is sufficient to greatly reduce the time for diffusion. In all experimental trials, the clustered network spread the message faster than the random network. This stands in contrast to the theory that networks with long-range ties, rather than redundancies, will result in faster diffusion. The participants in these experiments were significantly more likely to exhibit increased participation in the social network after receiving multiple copies of messages. While the study was highly controlled and idealized, the results suggest that there is still a great deal to learn about modeling real-world social networks. **KHG**

Some Properties of Cyclic Compositions, Arnold Knopfmacher and Neville Robbins. *The Fibonacci Quarterly* 48:3 (August 2010) 249–255.

A composition of n into k parts is a sequence $(a_1, a_2, \ldots, a_k)$ of k positive integers whose sum is n. There are $\binom{n-1}{k-1}$ such compositions. Say that two of the corresponding circular sequences are equivalent if they differ only by a circular shift. The number of equivalence classes is denoted by $\left\langle {n \atop k} \right\rangle$. The table of values of $\left\langle {n \atop k} \right\rangle$ for $n \geq 2$ and $1 \leq k \leq n-1$,

$$\begin{array}{cccccccc}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 2 & 2 & 1 \\
1 & 3 & 4 & 3 & 1 \\
1 & 3 & 5 & 5 & 3 & 1 \\
1 & 4 & 7 & 10 & 7 & 4 & 1 \\
1 & 4 & 10 & 14 & 14 & 10 & 4 & 1,
\end{array}$$

shares some features with Pascal's triangle. The appearance of the Fibonacci sequence in Pascal's triangle is well known. Here we see that every other row of the $\left\langle {n \atop k} \right\rangle$ table has a repeated Catalan number: 1, 2, 5, 14, ...

The authors prove the symmetry property $\left\langle {n \atop n-k} \right\rangle = \left\langle {n \atop k} \right\rangle$ for $1 \leq k \leq n-1$ and that $\left\langle {2n \atop n} \right\rangle$ is divisible by 2. Analytic and combinatorial methods are used to obtain the connection between cyclic composition numbers and binomial coefficients, $\left\langle {n \atop k} \right\rangle = \frac{1}{n}\binom{n}{k}$ if $(n, k) = 1$. The analytic approach is based on the bivariate generating function

$$C(z, u) = \sum_{n>0}\sum_{k\geq 0} \left\langle {n \atop k} \right\rangle z^n u^k = \sum_{j\geq 1} \frac{\phi(j)}{j} \log\left(1 - \frac{z^j u^j}{1-z^j}\right)^{-1},$$

which has as a consequence the formula

$$\left\langle {n \atop k} \right\rangle = [z^n u^k] C(z, u) = \frac{1}{n} \sum_{j|(n,k)} \phi(j) \binom{n/j}{k/j},$$

where ϕ is the Euler totient function. The authors give two proofs of the symmetry property: (i) by means of the explicit formula, and (ii) using a bijection between the necklaces with n beads, k of which are black and $n-k$ white, and the cyclic compositions of n into k parts. **CR**

Impact of Proof Validation on Proof Writing in Abstract Algebra, Robert A. Powers, Cathleen Craviotto, and Richard M. Grassi. *International Journal of Mathematical Education in Science and Technology* 41:4 (2010) 501–514.

The authors report an attempt to improve students' proof-writing skills in an undergraduate abstract algebra course by having them validate mathematical arguments. Forty students, many of them preservice secondary teachers, were divided into two sections, one taught with a proof-validation activity once a week, and the other served as a control group. Both sections were taught in a semi-interactive manner, incorporating lectures "enhanced by questions and answers" and cooperative group work. In the proof validation section, approximately 20–25 minutes were spent every Friday on validation activities during which students worked in small groups to analyze a mathematical argument, decide whether it was a proof, justify their claim, and report back to the class. (All 15 validation activities are given in an appendix.)

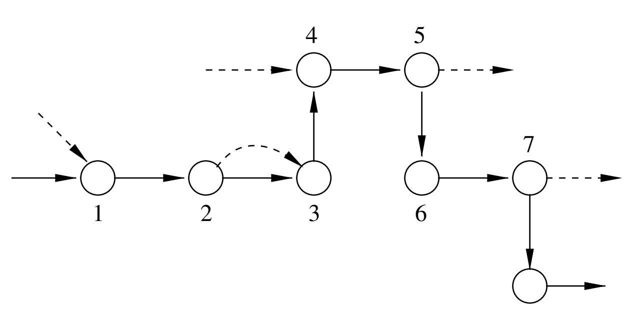
Statistical tests done on the three exams and the final (also included in the article) showed that those in the validation section improved modestly more than those in the control section. Nevertheless, the authors feel that "proof validation is a valuable objective . . . in and of itself" and they have decided to incorporate validation activities in sections of linear algebra and real analysis. **A&JS**

Massively Collaborative Mathematics, Julie Rehmeyer. *SIAM News* 43:3 (April 2010) 1–3.

This article tells a remarkable story involving Cambridge University mathematician and Fields Medalist Timothy Gowers. Gowers used his blog, `http://gowers.wordpress.com`, to direct more than two dozen mathematicians in collaborative proof of the density Hales-Jewett theorem. This theorem addresses the question of how many squares would have to be removed from a multidimensional tic-tac-toe board of arbitrary size to make it impossible for either player to win. The theorem has widespread importance in mathematics and theoretical computer science. It plays a particularly important role in Ramsey theory, additive combinatorics, and the "parallel repetition problem" in computer science. Gowers had an approach in mind for proving the theorem, but he had not yet been able to make the approach work. He therefore invited the mathematical world to collaborate through his blog, where he wrote, "The hard thought would be done by a sort of super-mathematician whose brain is distributed amongst bits of the brains of lots of interlinked people." Within days, mathematicians began to post suggestions. And within six weeks, a beautiful proof was completed that went further than Gowers had expected. The proof will be published under the name "D.H.J. Polymath." **RNG**

Adding Nesting Structure to Words, Rajeev Alur and P. Madhusudan. *Journal of the ACM* 56:3 (May 2009) 1–43.

The authors introduce a new formalism, called *nested words*, that can be used to model data with both linear and hierarchical structures, such as arithmetic expressions, computer programs, web documents, and various types of binary trees. In addition to a linear sequence of symbols, a nested word contains *nested edges*, which are noncrossing directed edges that connect pairs of symbols. Edges with the source or target vertex missing are also allowed. In the nested word below, a linear sequence of eight symbols (not shown) is enhanced by five nested edges $(-\infty, 1)$, $(-\infty, 4)$, $(2, 3)$, $(5, \infty)$, $(7, \infty)$.



The authors establish a bijection between nested words and strings enhanced with the tags "$\langle$" and "$\rangle$", where each symbol a has three flavors: a, $\langle a$, and $a\rangle$. The source vertex a of a nested edge corresponds to $\langle a$ and the target vertex to $a\rangle$. For example, assuming all symbols are a, the corresponding linear encoding of the above nested word is $a\rangle\langle aa\rangle a\rangle\langle aa\langle aa$. Since the edges are non-crossing, the corresponding tags are *properly* nested, that is, for any two pairs of $\langle\ \rangle$, one must be completely inside or completely outside the other.

The article extends the classical concept of *regular languages* to nested words by using finite automata enhanced with nested edges along which additional states may propagate. As in the classical setting, regular languages of nested words have equivalent formulations in terms of formal grammars and monadic second-order logic. Linearly encoded regular languages are called *visibly pushdown languages*. The set of all properly nested tagged words (as described above) is such an example. The article's main results show that visibly pushdown languages form a proper subclass of classical context-free languages. Visibly pushdown languages are closed under Boolean operations, homomorphism, concatenation, reversal, and other word operations, and there exist efficient algorithms to determine their emptiness, equivalence, and inclusion decision problems. These properties lead to new efficient algorithms for software verification that were not possible previously.

Similar closure and decision results are shown for infinite nested words. The authors also explore relationships between nested words and other models (e.g., regular nested words, which are exponentially more concise over tree automata in describing languages). **NT**

Moody's Mega Math Challenge Winning Paper: Making Sense of the 2010 Census, A. Das Sarma, J. Hurwitz, D. Tolnay, and S. Yu (Montgomery Blair High School, Silver Spring, MD). *SIAM Undergraduate Research Online* (SIURO) 3 (August 4, 2010) 159–175.

Sponsored by The Moody's Foundation and organized by SIAM, The M^3 Challenge is an annual mathematical modeling competition for teams of 11th and 12th grade students. These students are given 14 hours to analyze an applied problem (with no outside help except for the Internet) and submit a solution subject to certain criteria. The 2010 M^3 Challenge problem required the students to analyze U.S. Census Bureau data and methodology, and propose mathematical recommendations for undercount adjustment, the best apportionment method for the U.S. House of Representatives, and the fairest way to draw Congressional districts. After several rounds of judging by professional mathematicians, a tentative rank of the best six papers was made from the 531 initial submissions. The final ranking of the teams was based on their oral presentation of their paper to a panel of five judges at the Moody Foundation's headquarters. The above cited article is an enhanced version of the winning paper written during the 14-hour period of the competition.

For the undercount problem, the winning team proposed examining public records to estimate the values of missing data from different segments of the population. For the House apportionment problem, the team evaluated six historical methods (Hill, Dean, Webster, Adams, Jefferson, and Hamilton-Vinton) and indicated why the Hamilton-Vinton method was best. In their analysis of Congressional districting, the team recommended that states divide their congressional districts impartially according to population density.

Complete information on the competition is available at `http://m3challenge.siam.org`. A description of the 2008 M^3 Challenge and its winning paper "Ethanol: Not All It Seems To Be" appeared in the January 2008 issue of this JOURNAL. **HJR**

Does Sea Level Change When a Floating Iceberg Melts?, Boon Leong Lan. *The Physics Teacher* 48 (May 2010) 328–329.

Archimedes' Principle states that an object immersed in water is buoyed up by a force that is equal to the weight of water it displaces. This implies that when an ice cube floating in a glass of water melts, the height of the water in the glass will not change. However, there is an implicit assumption here that the densities of the water and the object are equal. In the case of an iceberg melting in the ocean, the density of seawater ($1024\,kg/m^3$) and of an iceberg ($1000\,kg/m^3$ are sufficiently different (icebergs are fresh water with no salt) so the sea level will not remain constant. This article offers a simple analysis, using algebra, to show that Archimedes' Principle predicts a slight rise in sea level when an iceberg melts. Archimedes' Principle says that the density of seawater times the volume of iceberg that is under water equals the density of the iceberg times the volume of the entire iceberg. When the iceberg melts, the mass of the resulting water is the same as the mass of the original iceberg. After the iceberg melts, sea level will have a rise of 0.021 times the volume of the original iceberg divided by the surface area of the ocean. The factor 0.021 depends only on the ratio between the densities of seawater and iceberg. **TL**

An Unexpected Use of Primes: Solving Sudokus by Calculator, Mark Spahn, Ron Lancaster, Deborah Moore-Russo, and Gerald Rising. *The Mathematical Gazette* 94:530 (July 2010) 224–232.

The authors show how Sudoku puzzles can be solved by programs written for the TI 84 calculator, which they make available free to the public at: `http://www.ascu.buffalo.edu/~insrisg/InsideYourCalculator.htm`. The user enters the starting numbers into a program that stores them in a matrix, putting a 0 in all blank cells. A second program, which searches for a solution, changes each 0 to 1, and each number n from 1 to 9 into the nth prime. Checking whether a number k between 1 and 9 is already in any *ennead* (a row, column, or 3 by 3 box) is accomplished by checking whether the product of the numbers in that ennead is divisible by k. The program contains three algorithms and an optional fourth. The first algorithm goes through all 81 cells to see if the contents of a blank cell can be determined. The second algorithm goes through each ennead to see if any cell in that ennead can be determined. The third algorithm looks for cells that exclude all but two possible digits, noting what happens with the first two algorithms if either of those digits is entered into the cell. The fourth algorithm is usually turned off because it is rarely needed and slows down the execution. For the most difficult Sudokus, in which the third algorithm fails, the program displays the message "HVPS FAIL'D. TOO BAD." On the rare occasion when this happens, the fourth algorithm can then be implemented to test pairs of alternatives for enneads. This will solve all but the most pathological Sudokus. **RNG**

An Integrated Modeling Environment to Study the Coevolution of Networks, Individual Behavior, and Epidemics, Chris Barrett, Keith Bisset, Jonathan Leidig, Achla Marathe, and Madhav Marathe. *AI Magazine* (Spring 2010) 75–87.

Agent-based modeling (ABM) involves simulating the interactions of many individuals in order to study the emergent patterns. ABM works at the individual level, tracking the actions and interactions of each individual ("agent") in the population. It allows each agent to act according to its own set of rules (and even to alter those rules in response to interactions). Differential equations-based modeling works at the population level, treating the number of people in each subpopulation (infected vs susceptible, for example) as variables, and typically assuming uniform behavior and universal contact. Differential equations-based models have the advantage of being mathematically tractable, whereas agent-base models can be very difficult to analyze and typically involve numerical simulation that requires massive computing power. However, agent-based models offer realistic simulations to study, as for example, the effect of interventions, public policy, and heterogeneous individual characteristics on an HIV or H1N1 epidemic.

This article describes ABM software that accepts up to 300 million individuals who are geographically located, and whose behaviors are based on real data. It also incorporates various networks, such as social contact and communication networks, that govern interactions between agents. To illustrate the promise of ABM for improving public policy, the authors use a case study of an influenza-like illness and examine the efficacy of interventions such as vaccination of certain subpopulations (children and health care workers), school closure, and self-isolation of exposed adults.

Given the extreme complexity of agent-based models, comparing simulations to field data and assessing the structural validity of the model is a crucial and challenging task. The article discusses and analyzes the advances and challenges in ABM. Although agent-based modeling is in an early state of development, rapid progress is being made. It will likely prove to be a valuable tool for improving public policy decisions in complicated situations, such as a looming epidemic. **TL**

Winning Odds, Yutaka Nishiyama and Steve Humble. *Plus 55* (June 2010).

A television show's magician game inspired this lucid account of nontransitivity in an elementary coin toss game. About the simplest example of nontransitivity is the scissors-rock-paper game: Scissors cuts paper (beating paper), paper covers rock (beating rock), and rock breaks scissors (beating scissors), yet transitivity would imply scissors beats rocks. In the coin game, each player selects a triplet of heads and tails, and the player whose triplet first comes up in repeated tosses is the winner. No matter which triplet one player selects, the second player can choose a triplet that is *more* likely to occur first. For an extreme case where one player chooses HHH, the other player can choose THH and will be far more likely to win. Less obviously, the choice THH is beaten by TTH. In fact, every one of the eight possible choices of a triplet can be beaten by an appropriate choice for the second player. Sample probabilities are derived and algorithms are given for choosing the superior triplet. There are historical references, especially to Martin Gardner, who discussed nontransitivity in a *Scientific American* column and, later, in his book *Time Travel and Other Mathematical Bewilderments*. This enticing and lively account is at `http://plus.maths.org/content/issue/55`. **NS**

Recognizing Graph Theoretic Properties with Polynomial Ideals, J. De Loera, C. Hillar, P. Malkin, and M. Omar. *The Electronic Journal of Combinatorics* 17:1 (2010).

The authors are interested in expanding on the *polynomial method* (a term coined by Noga Alon) to study three notoriously difficult problems in graph theory: k-colorability, Hamiltonicity, and determining the automorphism group of a graph. In each case, the strategy is to associate with a combinatorial question (for instance: "Is this graph 3-colorable?") a system of polynomial equations so that the answer to the combinatorial question is equivalent to determining if the system of polynomial equations has a solution. There is a (previously known) way to translate 3-colorability into a polynomial formulation.

A graph G is 3-colorable if and only if the system of equations described by

$$J_G = \{x_i^3 - 1 = 0, x_i^2 + x_i x_j + x_j^2 = 0 : i \text{ is a vertex of } G,\ \{i, j\} \text{ is an edge of } G\}$$

has a common zero over the algebraic closure over a field K with characteristic relatively prime to 3. If this system of equations does not have a common zero over the finite field of order 2, then the graph is not 3-colorable.

The authors are able fully to translate from the lack of a common zero to a combinatorial condition on graphs given by an existence condition for oriented 3- and 4-cycles in the graph G that satisfy certain properties. Furthermore, their combinatorial existence condition can be checked in polynomial time, giving an efficient running time for a solution to a hard combinatorial recognition problem. This leap from system of equations back to a combinatorial characterization that can be verified in polynomial time is a result of the power of this nonlinear polynomial method.

The flavor of the results in the rest of the paper are similar to the result about non-3-colorability. The article is a nice blend of theory and practical computation, and it contains a wealth of information and ideas. The article is available for free online at `http://www.combinatorics.org/Volume_17/PDF/v17i1r114.pdf` **CS**

Sets Versus Trial Sequences, Hausdorff Versus von Mises: "Pure" Mathematics Prevails in the Foundation of Probability Around 1920, Reinhard Siegmund-Schultze. *Historia Mathematica* 37:2 (May 2010) 204–241.

In 2004, the author discovered two letters in the Richard von Mises Papers in the Harvard University Archives that were written in late 1919, from "pure" mathematician Felix Hausdorff to von Mises. Von Mises had been trained as a mechanical engineer, but it was as an applied mathematician that he became the first director of the Institute for Applied Mathematics at the University of Berlin in 1920. In the previous year, von Mises's famous article "Foundations of the Calculus of Probability" criticized earlier attempts by Hausdorff and others who, ignoring intuitive notions of empirical probability like randomness, defined probability as "the quotient of the measure of a point set divided by the measure of the set in which that point set is contained." Von Mises proposed his own alternative foundation for a theory of probability, basing it not on sets and their Lebesgue measure but on limits of sequences of relative frequencies. In his two letters, Hausdorff criticized von Mises's attempt, in part because limits of relative sequences, when interpreted as set functions, lacked properties such as countable additivity (σ-additivity). Von Mises had used Carathéodory's notion of an outer measure and a measure function, that appeared in a book Carathéodory published in 1918, in the belief that a measure could be assigned to any set. But Hausdorff produced an infinite family of counterexamples to this claim. Siegmund-Schultze asserts, "The subtlety of Carathéodory's notion probably escaped von Mises's attention, or he found it irrevelant for applications." The author also notes, "von Mises's controversy with Pólya on the proof of the central limit theorem, which took place at about the same time as his discussion with Hausdorff ... showed that von Mises did not have full technical command of theorems on sequences of monotonic functions." Siegmund-Schultze discusses this controversy in his article "Probability in 1919/20: the von Mises-Pólya Controversy," *Archive for History of Exact Sciences* 60:5 (September 2006) 431–515.

According to the author, Hausdorff had a pure mathematician's misunderstanding of von Mises's intent, and, "Generally speaking, Hausdorff seems to have taken von Mises too literally by his words (such as 'measure function') ..." As a result of Hausdorff's second letter, von Mises published a "correction" in 1920, that dropped his claim of σ-additivity. But von Mises stuck to his two postulates for probability theory, even as the measure-theoretic view gained prevalence in the 1920s and 1930s. This view culminated in 1933 with Kolmorgoroff's booklet axiomatizing probability theory based on abstract measure theory. John von Neumann originally adopted von Mises's approach to probability in his quantum mechanics, but "von Mises's empiricist standpoint with respect to probability theory did not go down well with the Göttingen mathematicians, among them famously Hermann Weyl." It was only late in his life, in the 1950s, that von Mises admitted the failure of his frequentist attempt to rigorously base probability theory on sequences instead of sets. This provides an example of the complicated, historical relation between "real world probability" and "mathematical probability." For more information about this relation, the author refers to J. L. Doob's article "The Development of Rigor in Mathematical Probability, (1900–1950)," pp 157–170 in *Development of Mathematics 1900–1950*, Pier, J. P. (Ed.), Birkhäuser, Basel, Boston, Berlin, 1994. **PR**

Media Correspondents

KHG Kris H. Green; **RNG** Raymond N. Greenwell; **TL** Tanya Leise; **PR** Peter Ross; **CR** Cecil Rousseau; **HJR** Henry J. Ricardo; **A&JS** Annie and John Selden; **NS** Norton Starr; **CS** Chris Storm; **PDS** Philip D. Straffin; **NT** Nicholas Tran.

Editor's Note. In July 2010, Rokicki, Kociemba, Morley Davidson, and John Dethridge used the methods described in "Twenty-Two Moves Suffice for Rubik's Cube," (September 2010 Media Highlights) to prove that twenty moves suffice, thereby solving the problem completely. See `http://www.cube20.org`.

